Algorithm Problem Solving (APS): Divide-and-Conquer

Niema Moshiri
UC San Diego SPIS 2019

What is an **algorithm**?

Goals of Algorithm Problem Solving (APS)

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- Emphasis on writing solutions **precisely** and **coherently**
- Practice **discovering** algorithms and **describing** them
- **Analyze** algorithms

Example: The Largest Integer Problem

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7	2	0	4	-9	5	1	-4	3	8	-2	-7	...	-1	-8	6	-3	-6	9	-5	2

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Let's solve the problem!

Easier Example: The Peanut Butter & Jelly Problem

1. Open **bag_toast**
2. Remove 2 pieces of toast **x** and **y** from **bag_toast**
3. Close **bag_toast**
4. Open **jar_pb**
5. Insert **knife** into **jar_pb**
6. Remove **knife** from **jar_pb**
7. Spread **knife** onto **x**
8. Wipe **knife**
9. Close **jar_pb**
10. ...

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- **APS** is the process of discovering the algorithm in the first place

APS ➔ Algorithm ➔ Program

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- **Input:** A list of integers ***ints***
- **Output:** An integer x in ***ints*** such that, for all integers y in ***ints***, $x \geq y$
  - In other words, x is a largest integer in ***ints***

Let’s solve the problem!

Example: The Largest Integer Problem

```
Algorithm largest_number(ints):
    x ← negative infinity
    For every integer y in ints:
        if y > x:
            x ← y
    Return x
```

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- Our algorithm is correct (can you prove it?)
- However, a single “person” has to look at every integer
- Even if we had more “people,” they have no way of helping
- Can we think of a way to speed things up by working in parallel?

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- Typically composed of two “types” of cases:
    - **Base Case:** Can be solved directly
    - **Recursive Case:** Can be solved using solutions of subproblems

Example: Counting People Recursively

```
Algorithm num_people(person):
    If person is at the front of the line:
        Return 1
    Else:
        neighbor ← the person in front of person
        Return num_people(neighbor) + 1
```

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- **Divide** a given problem into several subproblems
- **Solve** each subproblem recursively
- **Combine** the solutions of the subproblems to solve the problem
- Tip: Try to balance the sizes of the subproblems as much as possible

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5. Analyze the algorithm
6. Write the solution
7. Revise

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- **Input:** A list of integers ***ints***
- **Output:** An integer $\boldsymbol{x}$ in ***ints*** such that, for all integers $\boldsymbol{y}$ in ***ints***, $\boldsymbol{x} \geq \boldsymbol{y}$
  - In other words, $\boldsymbol{x}$ is a largest integer in ***ints***

Let’s solve the problem!

Example: The Largest Integer Problem

0	1	2	3	4	5	6	7
a	b	c	d	e	f	g	h

```
largest_integer(ints, start, end)
```

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0	1	2	3	4	5	6	7
a	b	c	d	e	f	g	h

```
largest_integer(ints,  0 ,  7 )
```

Example: The Largest Integer Problem

0	1	2	3	4	5	6	7
a	b	c	d	e	f	g	h

```
largest_integer(ints,  0 , 3 )
```

Example: The Largest Integer Problem

0	1	2	3	4	5	6	7
a	b	c	d	e	f	g	h

```
largest_integer(ints,  0 ,  1 )
```

Example: The Largest Integer Problem

0	1	2	3	4	5	6	7
a	b	c	d	e	f	g	h

```
largest_integer(ints, 0 , 0 )
```

Example: The Largest Integer Problem

0	1	2	3	4	5	6	7
a	b	c	d	e	f	g	h

```
largest_integer(ints, 0 , 0 )
```

a

Example: The Largest Integer Problem

0	1	2	3	4	5	6	7
a	b	c	d	e	f	g	h

```
largest_integer(ints,   1  ,  1 )
```

b

Example: The Largest Integer Problem

0	1	2	3	4	5	6	7
i		c	d	e	f	g	h

```
largest_integer(ints,  0 ,  1 )
```

```
i = max(a,b)
```

Example: The Largest Integer Problem

0	1	2	3	4	5	6	7
i		c	d	e	f	g	h

```
largest_integer(ints,   2  ,  3 )
```

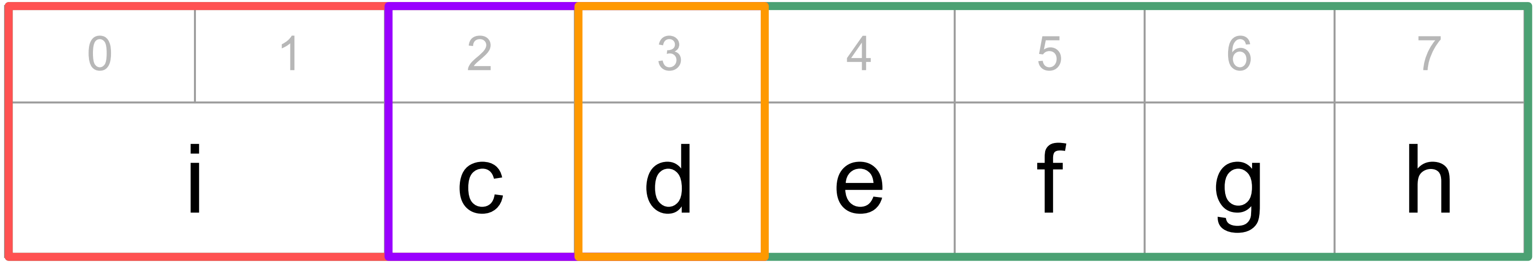
Example: The Largest Integer Problem

0	1	2	3	4	5	6	7
i		c	d	e	f	g	h

```
largest_integer(ints,   2  ,  2 )
```

```
c
```

Example: The Largest Integer Problem




```
largest_integer(ints,   3  ,  3 )
```

d

Example: The Largest Integer Problem

0	1	2	3	4	5	6	7
i		j		e	f	g	h

```
largest_integer(ints, 2 , 3 )
```

```
j = max(c,d)
```

Example: The Largest Integer Problem

<table>
<tr><th>0</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th></tr>
<tr><td colspan="4">k</td><td>e</td><td>f</td><td>g</td><td>h</td></tr>
</table>

```
largest_integer(ints,  0 ,  3 )
```

```
k = max(i,j)
```

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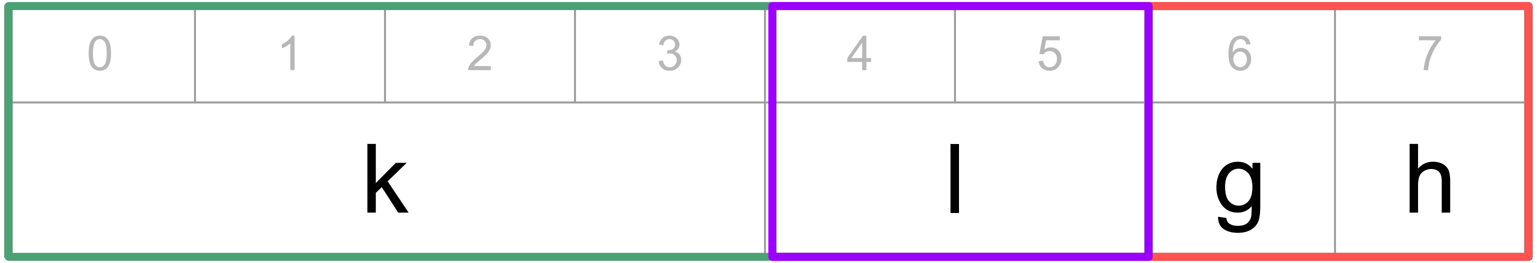


```
largest_integer(ints, 4 , 5 )
```

```
l = max(e,f)
```

Example: The Largest Integer Problem

0	1	2	3	4	5	6	7
k				l		m	

```
largest_integer(ints,  6 ,  7 )
```

```
m = max(g,h)
```

Example: The Largest Integer Problem

0	1	2	3	4	5	6	7
k				n			

```
largest_integer(ints,  4 ,  7 )
```

```
n = max(l,m)
```

Example: The Largest Integer Problem

0	1	2	3	4	5	6	7
n							

```
largest_integer(ints, 0 , 7 )
```

```
n = max(k,n)
```

Example: The Largest Integer Problem

```
Algorithm largest_number(ints, start, end):
    If start equals end:
        Return ints[start]
    Else:
        mid ← floor((start + end) / 2)
        left ← largest_number(ints, start, mid)
        right ← largest_number(ints, mid+1,
end)
        Return max(left, right)
```